An Economic "Kessler Syndrome": A Dynamic Model of Earth Orbit Debris

Nodir Adilov, Peter J. Alexander, and Brendan M. Cunningham

Current Draft: January 30, 2018

Abstract

The "Kessler Syndrome" (Kessler and Cour-Palais (1978)), refers to a scenario in which earth orbits inevitably become so polluted with satellite-related orbital debris that a self-reinforcing collisional cascade, which destroys satellites in orbit and makes orbital space unusable, is inevitable. We suggest that an "economic Kessler Syndrome," in which orbital debris renders orbits economically unprofitable, may precede a physical Kessler Syndrome. Our model predicts orbital debris increases when the rate of orbital decay or the level of orbital debris is low, and we suggest the rate of satellite launches is non-monotonically related to the level of orbital debris.

*Adilov: Indiana University-Purdue University Fort Wayne; Alexander: Federal Communications Commission, Washington DC; Cunningham: Eastern Connecticut State University. Adilov and Cunningham gratefully acknowledge funding in support of this project from a NASA Connecticut Space Grant (PTE Federal Award No.: NNX15AI12H). Alexander used no government resources on this project. David Sappington made many helpful comments on this draft.

1 Introduction

In the movie *Gravity*, a Russian missile strike on a decommissioned satellite sets off a collisional cascade between the resultant satellite debris and functioning low-earth orbit satellites, destroying operational satellites, and rendering the orbital space unusable. The science behind the movie is based on work by Kessler and Cour-Palais (1978) who suggest that as debris from launch vehicles and damaged satellites accumulates in orbit, eventually a tipping point is reached where a collisional cascade becomes inevitable. This outcome is popularly referred to as the "Kessler Syndrome," and the National Academy of Sciences reports:

> ...the current orbital debris environment has already reached a "tipping point." That is, the amount of debris, in terms of the population of large debris objects, as well as overall mass of debris currently in orbit, has reached a threshold where it will continually collide with itself, further increasing the population of orbital debris. This increase will lead to corresponding increases in spacecraft failures, which will only create more feedback into the system, increasing the debris population growth rate. [1]

In what follows, we construct a dynamic economic model that suggests an "economic" Kessler Syndrome may precede a "physical" Kessler Syndrome, as firms find it economically unprofitable to launch new satellites even when there are no functioning satellites in orbit. We suggest that as the quantity of orbital debris increases, orbital space may become economically unprofitable before it becomes physically unusable. Specifically, our model predicts (1) the quantity of space debris will increase even in the absence of new satellite launches when the rate of orbital decay is relatively low, and, (2) launch rates respond non-monotonically to debris: at low levels of debris the relationship is positive and increasing, but at high levels there is a tipping point beyond which launches

[1] National Research Council (2011)

contract as debris continues to accumulate.

2 Related Literature

The economic literature on orbital debris is sparse, and our work represents the first dynamic economic model to capture economic sustainability when launch rates interact with the quantity and rate of decay of orbital debris. Adilov et al. (2014) is in the spirit of this work, but in a two-period model. Muller et al. (2017) offer a multi-period analysis of satellite launches in the presence of debris. Unlike our effort, they assume (1) exogenous collision probabilities; (2) fixed revenue per satellite; and (3) no launch debris. Within this framework the authors find that satellite operators launch more satellites as debris increases (roughly the opposite of the Kessler effect). Klima et al. (2016) provide a model of debris removal in which two space agencies are engaged in a game and choose debris removal levels. Using a two-period model Macauley (2015) investigates a range of policy solutions to debris generation.

3 Model

Assume there is a competitive market providing satellite services, with the associated linear inverse demand function:

$$P_t = a - bS_t \tag{1}$$

where t is an index of time, P_t is price, and S_t is the number of functioning satellites at period t. The total number of satellite launches in period t is L_t. Assume a satellite launch has a constant marginal cost, $h > 0$, and the level of marginal cost of operating a satellite is given by $c > 0$ for a functioning satellite. Finally, assume that $a - c > 0$.

The discount factor is given by β. We assume satellites depreciate at a constant rate δ, where $0 \leq \delta \leq 1$, and depreciated satellites are retired in Earth's atmosphere upon re-entry (in the case of low-earth orbit) or relocated to a "graveyard orbit" (in the case of geostationary satellites).

Let D_t denote the quantity of orbital debris at time t. We assume orbital debris decays at a rate of ϕ, where $0 \leq \phi \leq 1$. Each period, γL_t units of new debris is generated from expended launch vehicles.

Collisions with satellites generate additional orbital debris. We assume a fraction, wD_t, of functioning satellites are hit by debris each period. We think of wD_t as the probability that a satellite might be hit by debris, and this probability increases proportionally with the quantity of debris. We assume a satellite hit by debris becomes non-functioning, and each collision between a satellite and debris generates n units of additional debris. Given this, we write the law of motion for orbital debris as:

$$D_{t+1} = (1 - \phi)D_t + \gamma L_t + wnD_t((1 - \delta)S_{t-1} + L_t) \tag{2}$$

while the number of functioning satellites in period t is given by:

$$S_t = (1 - wD_t)((1 - \delta)S_{t-1} + L_t) \tag{3}$$

Under the assumptions above, orbital space is *physically unusable* at period t if $D_t \geq \frac{1}{w}$, because all functioning satellites will be hit during the period. Thus, $D_t \geq \frac{1}{w} = D^{Kessler}$ is an expression of the "Kessler Syndrome," in which a collisional cascade renders an earth orbit unusable.

Next, we solve the model for the equilibrium number of launches and describe the rate of debris generation. We assume a firm launches a satellite as long as its expected marginal revenue from the launch is not smaller than its expected marginal cost. Expected marginal revenue equals the sum of discounted expected revenue streams from current and future periods and expected marginal cost equals the sum of the firm's discounted expected marginal costs from current and future periods.

To keep the model analytically tractable, firms have adaptive expectations regarding future price levels and probabilities of being hit by orbital debris.[2] In this case, expected marginal revenue from a launch in period t is equal to $(1 - wD_t)P_t(1 + \beta(1 - wD_t)(1 - \delta) + ...) = \frac{(1-wD_t)P_t}{1-\beta(1-wD_t)(1-\delta)}$, while expected marginal cost equals to $h + (1 - wD_t)c(1 + \beta(1-wD_t)(1-\delta)+...) = h+\frac{(1-wD_t)c}{1-\beta(1-wD_t)(1-\delta)}$. In equilibrium, expected marginal revenue equals expected marginal cost, i.e., $\frac{(1-wD_t)P_t}{1-\beta(1-wD_t)(1-\delta)} = h + \frac{(1-wD_t)c}{1-\beta(1-wD_t)(1-\delta)}$. Substituting P_t into this equality yields the equilibrium number of satellites and launches in period t:

$$S_t = \frac{(a-c)}{b} - \frac{(1-\beta(1-wD_t)(1-\delta))h}{b(1-wD_t)} \tag{4}$$

$$L_t = \frac{S_t}{1-wD_t} - (1-\delta)S_{t-1} \tag{5}$$

We note that equation (4) assumes that $\frac{(a-c)}{b} - \frac{(1-\beta(1-wD_t)(1-\delta))h}{b(1-wD_t)} \geq (1-\delta)S_{t-1}(1-wD_t)$, i.e., the stock of satellites in period $t-1$ is not too large.

Proposition 3.1 $L_t = 0$ *if* $D_t \geq \frac{1}{w}(1 - \frac{h}{a-c+\beta(1-\delta)h})$.

The proofs of all propositions are given in the appendix. Proposition 3.1 implies that firms will find it *economically unprofitable* to launch satellites into orbit if the quantity of space debris is larger than a cutoff value (D^{Econ}) even if there there are no

[2]In a follow-on piece, we relax this assumption, and use simulations to solve the model.

functioning satellites in orbit (i.e., even when $S_{t-1} = 0$). Comparing this cutoff value to the cutoff value when space becomes physically unusable (the Kessler Syndrome), we see that space becomes economically unprofitable for lower levels of space debris ($D^{Kessler} = \frac{1}{w} > \frac{1}{w}(1 - \frac{h}{a-c+\beta(1-\delta)h}) = D^{Econ}$). In other words, as the quantity of space debris increases, space becomes economically unprofitable before it becomes physically unusable.

Next, we explore changes in space debris over time.

Proposition 3.2 *In equilibrium with $S_t > 0$ and $L_t > 0$, $D_{t+1} - D_t > 0$ if $\phi < \phi^*$ or if $D_t < D^*$ for some $\phi^* > 0$ and $D^* > 0$.*

The value of ϕ^* is given in the appendix. Proposition 3.2 implies that the quantity of orbital debris increases if the rate of debris decay or the quantity of debris is sufficiently low. Intuitively, for low rates of debris decay, firms generate more new debris than the debris that decays. Similarly, for low levels of orbital debris, launches are more profitable and firms launch more satellites and generate more debris than the quantity of decaying debris.

Proposition 3.3 *In equilibrium with $S_t > 0$ and $L_t > 0$:*

(i) $\frac{dS_t}{dD_t} < 0$ *and* $\frac{d^2S_t}{dD_t^2} < 0$,

(ii) $\frac{dL_t}{dD_t} > 0$ *and* $\frac{d^2L_t}{dD_t^2} > 0$ *when* $D_t < \frac{1}{w}(1 - \frac{3h}{a-c+\beta(1-\delta)h})$,

(iii) $\frac{dL_t}{dD_t} > 0$ *and* $\frac{d^2L_t}{dD_t^2} < 0$ *when* $D_t \in (\frac{1}{w}(1 - \frac{3h}{a-c+\beta(1-\delta)h}), \frac{1}{w}(1 - \frac{2h}{a-c+\beta(1-\delta)h}))$, *and*

(iv) $\frac{dL_t}{dD_t} < 0$ *and* $\frac{d^2L_t}{dD_t^2} < 0$ *when* $D_t > \frac{1}{w}(1 - \frac{2h}{a-c+\beta(1-\delta)h})$.

Proposition 3.3 suggests that for low levels of orbital debris the launch rate increases at an increasing rate, as firms try to replace lost satellites. As the quantity of debris

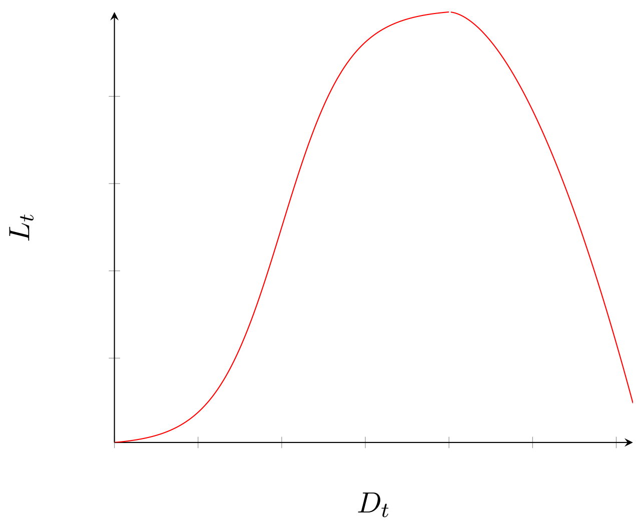

Figure 1: Launch Rates and Debris

increases further, the launch rate increases at a decreasing rate because a smaller fraction of satellites are being replaced. Above a certain threshold launches are decreasing in debris since the return to launching has declined. Figure 1 presents these results graphically.

The number of functioning satellites in orbit is a decreasing function of the quantity of space debris in equilibrium, keeping all else constant. This implies that increased launches do not fully compensate for lost satellites, and the quantity of functioning satellites decreases at an increasing rate.

Proposition 3.4 *In equilibrium where* $L_t = 0$ *and* $D_t > 0$, $D_{t+1} - D_t > 0$ *if* $\phi < wn(1 - \delta)S_{t-1}$.

Proposition 3.4 suggests that the quantity of debris may increase even if launches cease. This occurs when collisions of extant debris with extant satellites generates more additional debris than the quantity of debris that decays.

4 Conclusion

The Kessler Syndrome refers to a particular physical model of debris accumulation in orbital space. The predictions of the model are stark: orbital space will be rendered physically unusable at a future point in time, even given remediation. We propose that the explicit introduction of economic incentives suggests an alternative in which orbital space becomes economically unprofitable perhaps well before it becomes physically unusable; in short, an economic Kessler Syndrome. This result is given in Proposition 3.1.

Propositions 3.2-3.4 explicate the dynamics of the quantity of debris and rates of new launches. Propositions 3.2 and 3.4 show conditions under which debris increases. As described in proposition 3.3., launch rates of new satellites may increase at an increasing rate when the quantity of debris is low, but increasing.

We uncover a non-monotonic relationship between launch rates and the volume of debris, i.e. a tipping point beyond which launches contract as debris continues to accumulate. We speculate that in a model with endogenous prices and more forward-looking decision making, launch rates would generally be higher for a given volume of debris, but the non-monotonic reaction of the industry to debris should still obtain.

References

Adilov, N., P. J. Alexander, and B. M. Cunningham (2014). An economic analysis of earth orbit pollution. *Environmental and Resource Economics 60*(1), 81–98.

Kessler, D. J. and B. G. Cour-Palais (1978). Collision frequency of artificial satellites: The creation of a debris belt. *Journal of Geophysical Research: Space Physics 83*(A6), 2637–2646.

Klima, R., D. Bloembergen, R. Savani, K. Tuyls, D. Hennes, and D. Izzo (2016). Space debris removal: A game theoretic analysis. *Games 7*(3).

Macauley, M. K. (2015). The economics of space debris: Estimating the costs and benefits of debris mitigation. *Acta Astronautica 115*, 160–164.

Muller, C., O. Rozanova, and M. Urdanoz (2017). Economic valuation of debris removal. 68th International Astronautical Congress.

National Research Council (2011). *Limiting Future Collision Risk to Spacecraft: An Assessment of NASA's Meteoroid and Orbital Debris Programs.* Washington, DC: The National Academies Press.

5 Appendix

Proof to Proposition 3.1

Substituting (4) into (5) yields:

$$L_t = \frac{1}{b}((a - c + \beta(1-\delta)h)(1 - wD_t) - 1) - (1-\delta)(1-wD_t)S_{t-1} \tag{6}$$

Then, $L_t = 0$ for any $S_{t-1} \geq 0$ when $D_t \geq \frac{1}{w}(1 - \frac{h}{a-c+\beta(1-\delta)h})$.

Proof to Proposition 3.2

When $\phi = 0$, $D_{t+1} - D_t = -\phi D_t + \gamma L_t + wnD_t((1-\delta)S_{t-1} + L_t) = \gamma L_t + wnD_t((1-\delta)S_{t-1} + L_t) > 0$. Thus, there exists $\phi^* > 0$ such that $D_{t+1} - D_t > 0$ when $\phi < \phi^*$. Similarly, when $D_t = 0$, $D_{t+1} - D_t = \gamma L_t > 0$. Thus, there exists $D^* > 0$ such that $D_{t+1} - D_t > 0$ when $D_t < D^*$. We calculate the value for ϕ^* by substituting the value for L_t into $D_{t+1} - D_t = -\phi^* D_t + \gamma L_t + wnD_t((1-\delta)S_{t-1} + L_t) = 0$:

$$\phi^* = \frac{1}{D_t}(\gamma L_t + wnD_t((1-\delta)S_{t-1} + L_t) \Leftrightarrow$$

$$\phi^* = \frac{1}{D_t}(\frac{\gamma + wnD_t}{1 - wD_t}(\frac{a-c}{b} - \frac{(1 - \beta(1 - wD_t)(1-\delta))h}{b(1 - wD_t)}) - \gamma(1-\delta)S_{t-1}) \tag{7}$$

Proof to Proposition 3.3

$$\frac{dS_t}{dD_t} = -\frac{hw}{b(1 - wD_t)^2} < 0 \tag{8}$$

$$\frac{d^2 S_t}{dD_t^2} = -\frac{2hw}{b(1 - wD_t)^3} < 0 \tag{9}$$

$$\frac{dL_t}{dD_t} = \frac{w}{b}(1 - wD_t)^{-2}(a - c + \beta(1-\delta)h - 2h(1 - wD_t)^{-1}) \tag{10}$$

$$\frac{d^2L_t}{dD_t^2} = \frac{2w^2}{b}(1-wD_t)^{-3}(a-c+\beta(1-\delta)h-3h(1-wD_t)^{-1}) \tag{11}$$

Then, $a-c+\beta(1-\delta)h-2h(1-wD_t)^{-1} > a-c+\beta(1-\delta)h-3h(1-wD_t)^{-1} > 0$ if $D_t < \frac{1}{w}(1-\frac{3h}{a-c+\beta(1-\delta)h})$. This, in turn, implies that $\frac{dL_t}{dD_t} > 0$ and $\frac{d^2L_t}{dD_t^2} > 0$ when $D_t < \frac{1}{w}(1-\frac{3h}{a-c+\beta(1-\delta)h})$. For $D_t \in (\frac{1}{w}(1-\frac{3h}{a-c+\beta(1-\delta)h}), \frac{1}{w}(1-\frac{2h}{a-c+\beta(1-\delta)h}))$, $a-c+\beta(1-\delta)h-2h(1-wD_t)^{-1} > 0 > a-c+\beta(1-\delta)h-3h(1-wD_t)^{-1}$ and therefore $\frac{dL_t}{dD_t} > 0$ and $\frac{d^2L_t}{dD_t^2} < 0$. For $D_t > \frac{1}{w}(1-\frac{3h}{a-c+\beta(1-\delta)h})$, $0 > a-c+\beta(1-\delta)h-2h(1-wD_t)^{-1} > a-c+\beta(1-\delta)h-3h(1-wD_t)^{-1}$ and therefore $\frac{dL_t}{dD_t} < 0$ and $\frac{d^2L_t}{dD_t^2} < 0$.

Proof to Proposition 3.4

Let $L_t = 0$ for some $D_t > 0$. Then, $D_{t+1} - D_t = wn(1-\delta)S_{t-1} - \phi$. This implies that $D_{t+1} - D_t > 0$ if $wn(1-\delta)S_{t-1} > \phi$.